Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7354647755

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2023**

1 hour 50 minutes

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 3 pages.

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1 Find the set of values of x satisfying the inequality $\left|2^{x+1} - 2\right| < 0.5$, giving your answer to 3 significant figures. [4]

2 On an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 1 + 2\mathrm{i}| \leqslant |z|$ and $|z - 2| \leqslant 1$. [5]

3 The polynomial $2x^3 + ax^2 + bx + 6$, where a and b are constants, is denoted by $\mathrm{p}(x)$. When $\mathrm{p}(x)$ is divided by $(x + 2)$ the remainder is -38 and when $\mathrm{p}(x)$ is divided by $(2x - 1)$ the remainder is $\frac{19}{2}$.

Find the values of a and b. [5]

4 Solve the quadratic equation $(3 + \mathrm{i})w^2 - 2w + 3 - \mathrm{i} = 0$, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

5 Find the exact coordinates of the stationary points of the curve $y = \dfrac{\mathrm{e}^{3x^2-1}}{1 - x^2}$. [6]

6 **(a)** Show that the equation $\cot^2\theta + 2\cos 2\theta = 4$ can be written in the form

$$4\sin^4\theta + 3\sin^2\theta - 1 = 0.$$ [3]

(b) Hence solve the equation $\cot^2\theta + 2\cos 2\theta = 4$, for $0° < \theta < 360°$. [3]

7 The equation of a curve is $x^3 + y^2 + 3x^2 + 3y = 4$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{3x^2 + 6x}{2y + 3}$. [3]

(b) Hence find the coordinates of the points on the curve at which the tangent is parallel to the x-axis. [5]

8 The variables x and y satisfy the differential equation

$$\mathrm{e}^{4x}\frac{\mathrm{d}y}{\mathrm{d}x} = \cos^2 3y.$$

It is given that $y = 0$ when $x = 2$.

Solve the differential equation, obtaining an expression for y in terms of x. [7]

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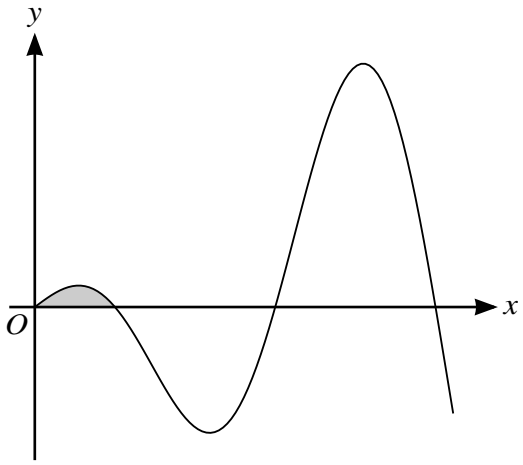
9 Let $\mathrm{f}(x) = \dfrac{17x^2 - 7x + 16}{(2 + 3x^2)(2 - x)}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^3. [5]

(c) State the set of values of x for which the expansion in **(b)** is valid. Give your answer in an exact form. [1]

10



The diagram shows the curve $y = x\cos 2x$, for $x \geqslant 0$.

(a) Find the equation of the tangent to the curve at the point where $x = \frac{1}{2}\pi$. [4]

(b) Find the exact area of the shaded region shown in the diagram, bounded by the curve and the x-axis. [5]

11 The line l has equation $\mathbf{r} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$. The points A and B have position vectors $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} - \mathbf{j} + \mathbf{k}$ respectively.

(a) Find a unit vector in the direction of l. [2]

The line m passes through the points A and B.

(b) Find a vector equation for m. [2]

(c) Determine whether lines l and m are parallel, intersect or are skew. [5]

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