Cambridge Assessment International Education

# Cambridge International AS & A Level

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| CANDIDATE NAME | | | | | | | | | |
| CENTRE NUMBER | | | | | | CANDIDATE NUMBER | | | |

*1087265794*

**MATHEMATICS** **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2022**

**1 hour 50 minutes**

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 4 pages.


© UCLES 2022

**1** **(a)** Sketch the graph of $y = |2x + 1|$. [1]

**(b)** Solve the inequality $3x + 5 < |2x + 1|$. [3]

**2** On a sketch of an Argand diagram shade the region whose points represent complex numbers $z$ satisfying the inequalities $|z| \leqslant 3$, $\mathrm{Re}\, z \geqslant -2$ and $\frac{1}{4}\pi \leqslant \arg z \leqslant \pi$. [4]

**3** Solve the equation $2^{3x-1} = 5(3^{-x})$. Give your answer in the form $\dfrac{\ln a}{\ln b}$, where $a$ and $b$ are integers. [4]

**4** Solve the equation $\tan(x + 45°) = 2\cot x$ for $0° < x < 180°$. [5]

**5** The complex numbers $u$ and $w$ are defined by $u = 2e^{\frac{1}{4}\pi i}$ and $w = 3e^{\frac{1}{3}\pi i}$.

**(a)** Find $\dfrac{u^2}{w}$, giving your answer in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the exact values of $r$ and $\theta$. [3]

**(b)** State the least positive integer $n$ such that both $\mathrm{Im}\, w^n = 0$ and $\mathrm{Re}\, w^n > 0$. [1]

**6** **(a)** Prove the identity $\cos 4\theta + 4\cos 2\theta + 3 \equiv 8\cos^4\theta$. [4]

**(b)** Hence solve the equation $\cos 4\theta + 4\cos 2\theta = 4$ for $0° \leqslant \theta \leqslant 180°$. [3]

**7** The equation of a curve is $y = \dfrac{x}{\cos^2 x}$, for $0 \leqslant x < \frac{1}{2}\pi$. At the point where $x = a$, the tangent to the curve has gradient equal to 12.

**(a)** Show that $a = \cos^{-1}\left(\sqrt[3]{\dfrac{\cos a + 2a\sin a}{12}}\right)$. [3]

**(b)** Verify by calculation that $a$ lies between 0.9 and 1. [2]

**(c)** Use an iterative formula based on the equation in part **(a)** to determine $a$ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

**8** In a certain chemical reaction the amount, $x$ grams, of a substance is increasing. The differential equation satisfied by $x$ and $t$, the time in seconds since the reaction began, is

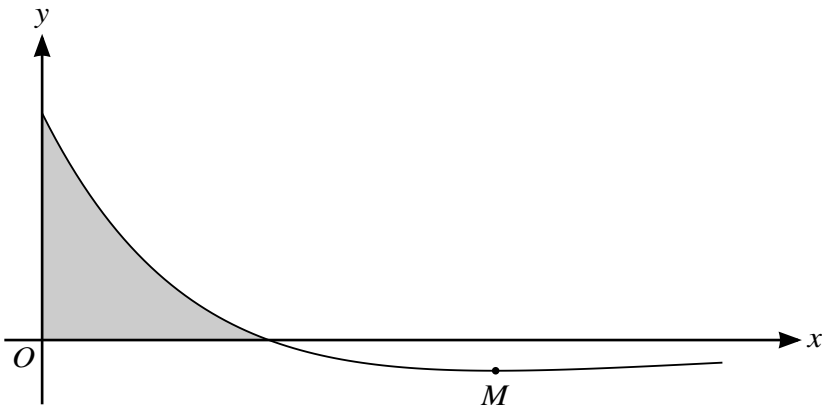
$$\frac{\mathrm{d}x}{\mathrm{d}t} = kx\mathrm{e}^{-0.1t},$$

where $k$ is a positive constant. It is given that $x = 20$ at the start of the reaction.

**(a)** Solve the differential equation, obtaining a relation between $x$, $t$ and $k$. [5]

**(b)** Given that $x = 40$ when $t = 10$, find the value of $k$ and find the value approached by $x$ as $t$ becomes large. [3]

**9**




The diagram shows part of the curve $y = (3 - x)\mathrm{e}^{-\frac{1}{3}x}$ for $x \geqslant 0$, and its minimum point $M$.

**(a)** Find the exact coordinates of $M$. [5]

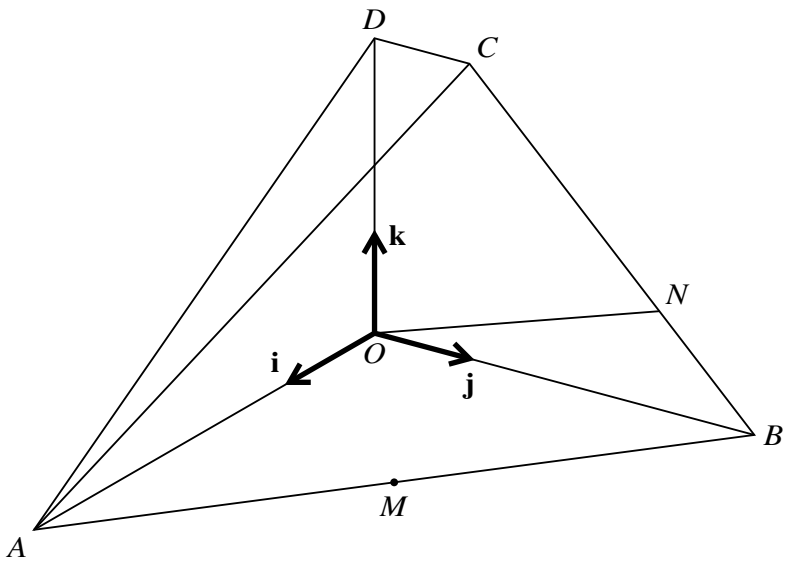
**(b)** Find the area of the shaded region bounded by the curve and the axes, giving your answer in terms of e. [5]

**10** Let $\mathrm{f}(x) = \dfrac{2x^2 + 7x + 8}{(1 + x)(2 + x)^2}$.

**(a)** Express $\mathrm{f}(x)$ in partial fractions. [5]

**(b)** Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of $x$, up to and including the term in $x^2$. [5]

**11**




In the diagram, $OABCD$ is a solid figure in which $OA = OB = 4$ units and $OD = 3$ units. The edge $OD$ is vertical, $DC$ is parallel to $OB$ and $DC = 1$ unit. The base, $OAB$, is horizontal and angle $AOB = 90°$. Unit vectors **i**, **j** and **k** are parallel to $OA$, $OB$ and $OD$ respectively. The midpoint of $AB$ is $M$ and the point $N$ on $BC$ is such that $CN = 2NB$.

**(a)** Express vectors $\overrightarrow{MD}$ and $\overrightarrow{ON}$ in terms of **i**, **j** and **k**. [4]

**(b)** Calculate the angle in degrees between the directions of $\overrightarrow{MD}$ and $\overrightarrow{ON}$. [3]

**(c)** Show that the length of the perpendicular from $M$ to $ON$ is $\sqrt{\frac{22}{5}}$. [4]

© UCLES 2022