Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER			

3967186601

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2023**

1 hour 50 minutes

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 6 pages.

© UCLES 2023

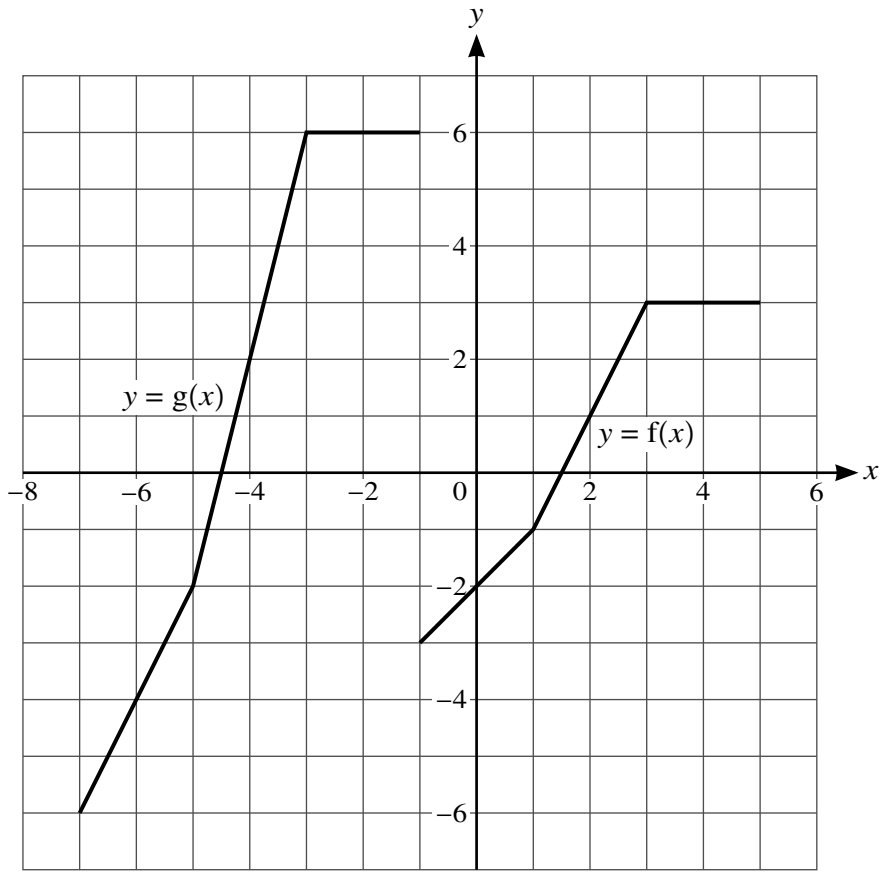
1 Solve the equation $4\sin\theta + \tan\theta = 0$ for $0° < \theta < 180°$. [3]

2 **(a)** Find the first three terms in the expansion, in ascending powers of x, of $(2 + 3x)^4$. [2]

(b) Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^5$. [2]

(c) Hence find the coefficient of x^2 in the expansion of $(2 + 3x)^4(1 - 2x)^5$. [2]

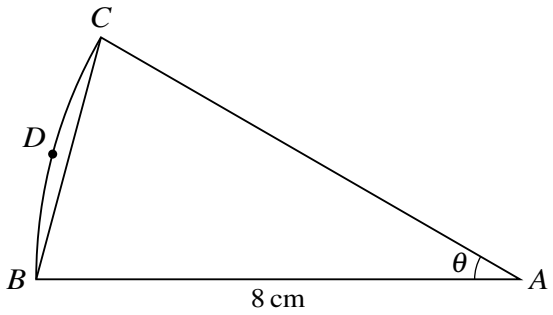
3



The diagram shows graphs with equations $y = f(x)$ and $y = g(x)$.

Describe fully a sequence of two transformations which transforms the graph of $y = f(x)$ to $y = g(x)$. [4]

4



The diagram shows a sector ABC of a circle with centre A and radius 8 cm. The area of the sector is $\frac{16}{3}\pi\,\text{cm}^2$. The point D lies on the arc BC.

Find the perimeter of the segment BCD. [4]

5 The line with equation $y = kx - k$, where k is a positive constant, is a tangent to the curve with equation $y = -\dfrac{1}{2x}$.

Find, in either order, the value of k and the coordinates of the point where the tangent meets the curve. [5]

6 The first three terms of an arithmetic progression are $\dfrac{p^2}{6}$, $2p - 6$ and p.

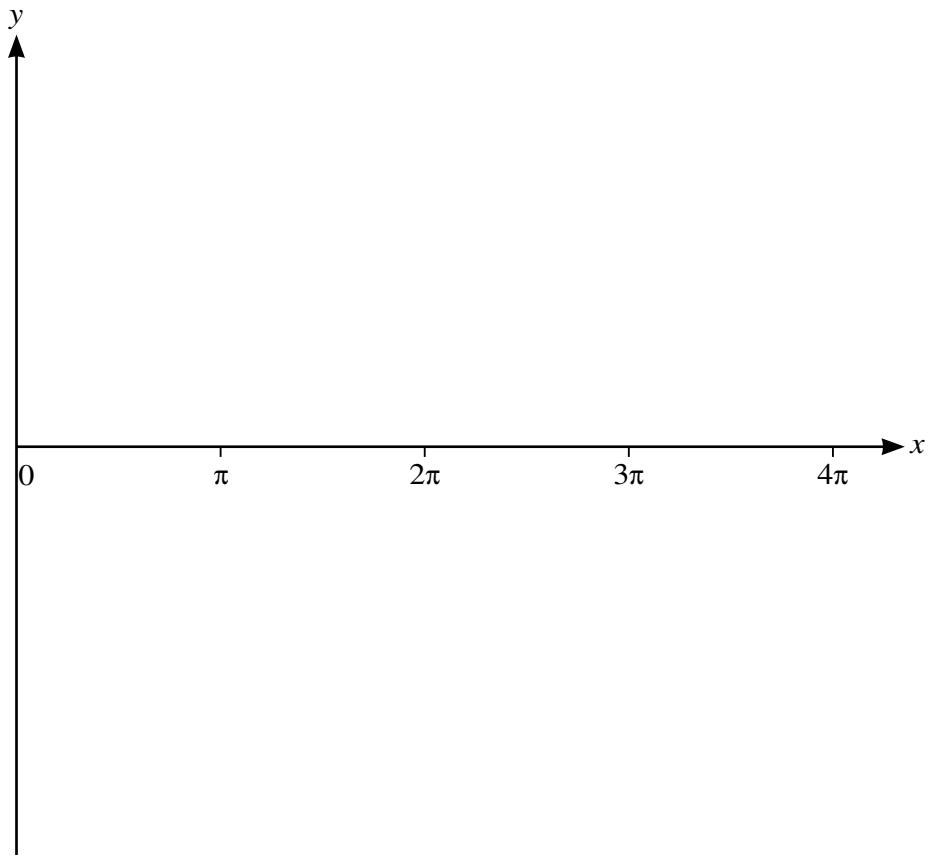
(a) Given that the common difference of the progression is not zero, find the value of p. [3]

(b) Using this value, find the sum to infinity of the geometric progression with first two terms $\dfrac{p^2}{6}$ and $2p - 6$. [2]

7 A curve has equation $y = 2 + 3\sin\frac{1}{2}x$ for $0 \leqslant x \leqslant 4\pi$.

(a) State greatest and least values of y. [2]

(b) Sketch the curve. [2]



(c) State the number of solutions of the equation

$$2 + 3\sin\tfrac{1}{2}x = 5 - 2x$$

for $0 \leqslant x \leqslant 4\pi$. [1]

8 The functions f and g are defined as follows, where a and b are constants.

$$\mathrm{f}(x) = 1 + \frac{2a}{x - a} \text{ for } x > a$$

$$\mathrm{g}(x) = bx - 2 \text{ for } x \in \mathbb{R}$$

(a) Given that $\mathrm{f}(7) = \frac{5}{2}$ and $\mathrm{gf}(5) = 4$, find the values of a and b. [4]

For the rest of this question, you should use the value of a which you found in **(a)**.

(b) Find the domain of f^{-1}. [1]

(c) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

© UCLES 2023

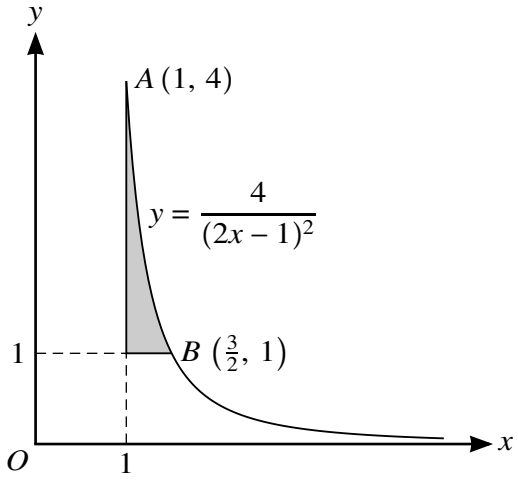
9 Water is poured into a tank at a constant rate of $500\,\text{cm}^3$ per second. The depth of water in the tank, t seconds after filling starts, is h cm. When the depth of water in the tank is h cm, the volume, $V\,\text{cm}^3$, of water in the tank is given by the formula $V = \frac{4}{3}(25 + h)^3 - \frac{62\,500}{3}$.

(a) Find the rate at which h is increasing at the instant when $h = 10$ cm. [3]

(b) At another instant, the rate at which h is increasing is 0.075 cm per second.

Find the value of V at this instant. [3]

10



The diagram shows part of the curve with equation $y = \dfrac{4}{(2x-1)^2}$ and parts of the lines $x = 1$ and $y = 1$. The curve passes through the points $A\,(1,\,4)$ and $B, \left(\frac{3}{2},\,1\right)$.

(a) Find the exact volume generated when the shaded region is rotated through 360° about the x-axis. [5]

(b) A triangle is formed from the tangent to the curve at B, the normal to the curve at B and the x-axis.

Find the area of this triangle. [6]

11 The equation of a curve is such that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 - 30x + 6a$, where a is a positive constant. The curve has a stationary point at $(a,\,-15)$.

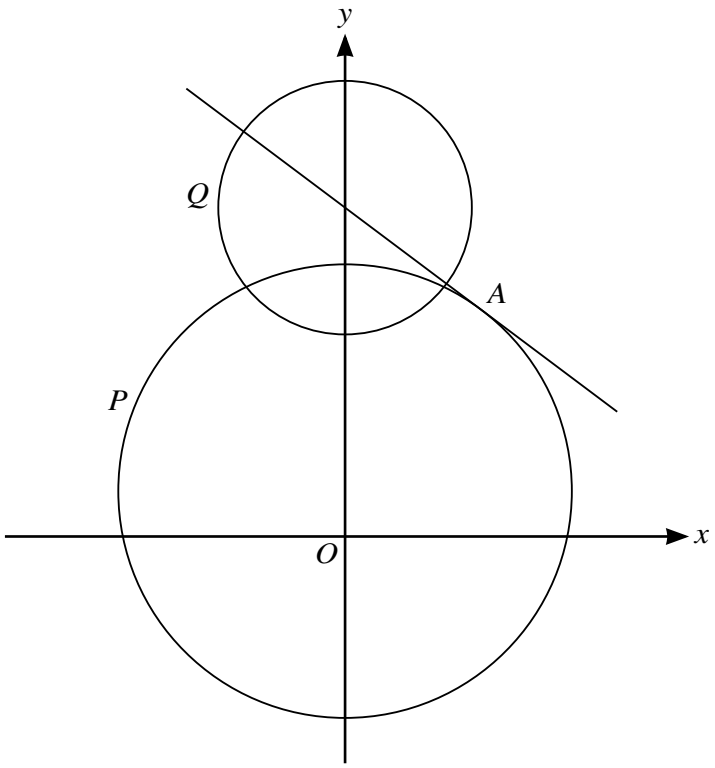
(a) Find the value of a. [2]

(b) Determine the nature of this stationary point. [2]

(c) Find the equation of the curve. [3]

(d) Find the coordinates of any other stationary points on the curve. [2]

12



The diagram shows a circle P with centre (0, 2) and radius 10 and the tangent to the circle at the point A with coordinates (6, 10). It also shows a second circle Q with centre at the point where this tangent meets the y-axis and with radius $\frac{5}{2}\sqrt{5}$.

(a) Write down the equation of circle P. [1]

(b) Find the equation of the tangent to the circle P at A. [2]

(c) Find the equation of circle Q and hence verify that the y-coordinates of both of the points of intersection of the two circles are 11. [3]

(d) Find the coordinates of the points of intersection of the tangent and circle Q, giving the answers in surd form. [3]

© UCLES 2023