Cambridge Assessment International Education

# Cambridge International AS & A Level

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| CANDIDATE NAME | | | | | | | | | |
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*7036013722*

**MATHEMATICS** **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2022**

**1 hour 50 minutes**

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

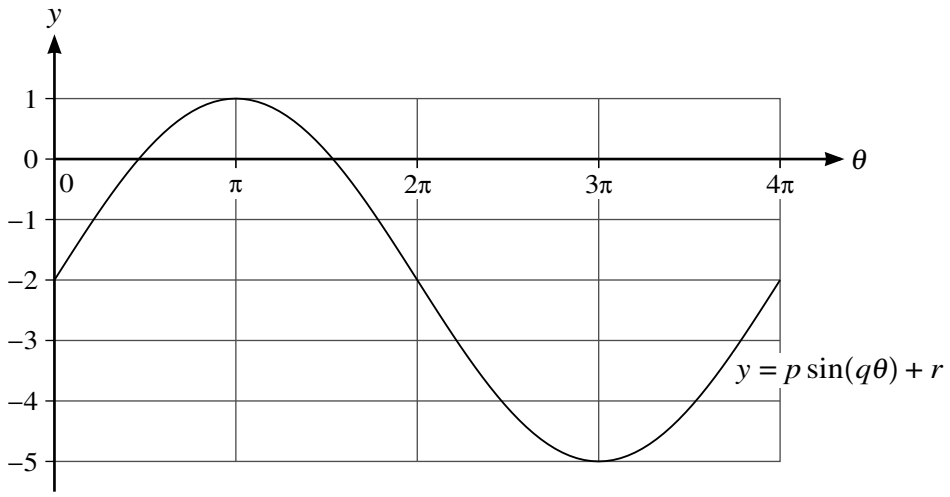
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**1** The coefficient of $x^3$ in the expansion of $\left(p + \frac{1}{p}x\right)^4$ is 144.

Find the possible values of the constant $p$. [4]

**2**




The diagram shows part of the curve with equation $y = p\sin(q\theta) + r$, where $p$, $q$ and $r$ are constants.

**(a)** State the value of $p$. [1]

**(b)** State the value of $q$. [1]

**(c)** State the value of $r$. [1]

**3** An arithmetic progression has first term 4 and common difference $d$. The sum of the first $n$ terms of the progression is 5863.

**(a)** Show that $(n-1)d = \frac{11726}{n} - 8$. [1]

**(b)** Given that the $n$th term is 139, find the values of $n$ and $d$, giving the value of $d$ as a fraction. [4]

**4** **(a)** The curve with equation $y = x^2 + 2x - 5$ is translated by $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

Find the equation of the translated curve, giving your answer in the form $y = ax^2 + bx + c$. [3]

**(b)** The curve with equation $y = x^2 + 2x - 5$ is transformed to a curve with equation $y = 4x^2 + 4x - 5$.

Describe fully the single transformation that has been applied. [2]

**5** **(a)** Solve the equation $6\sqrt{y} + \dfrac{2}{\sqrt{y}} - 7 = 0$. [4]

**(b)** Hence solve the equation $6\sqrt{\tan x} + \dfrac{2}{\sqrt{\tan x}} - 7 = 0$ for $0° \leqslant x \leqslant 360°$. [3]

**6** The function f is defined by $\mathrm{f}(x) = 2x^2 - 16x + 23$ for $x < 3$.

**(a)** Express $\mathrm{f}(x)$ in the form $2(x + a)^2 + b$. [2]

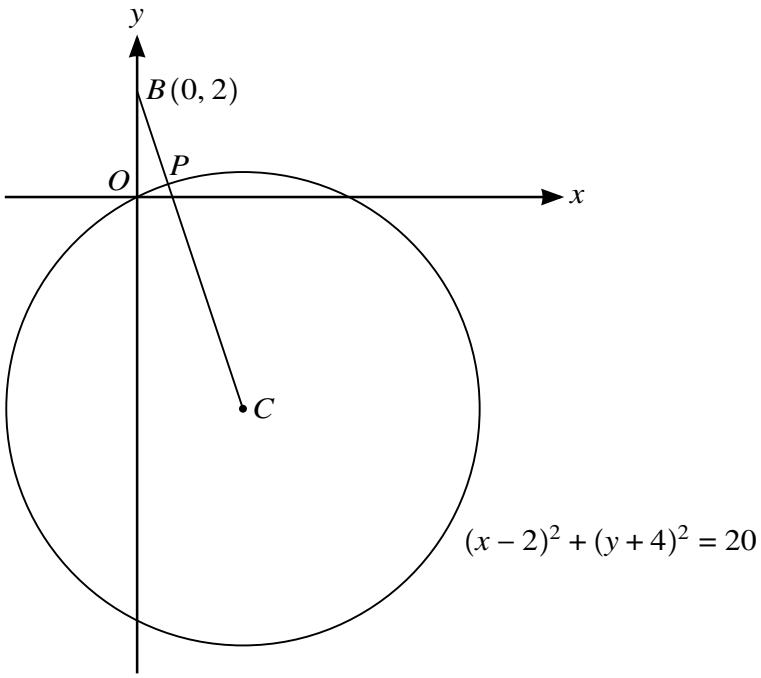
**(b)** Find the range of f. [1]

**(c)** Find an expression for $\mathrm{f}^{-1}(x)$. [3]

The function g is defined by $\mathrm{g}(x) = 2x + 4$ for $x < -1$.

**(d)** Find and simplify an expression for $\mathrm{fg}(x)$. [2]

**7**

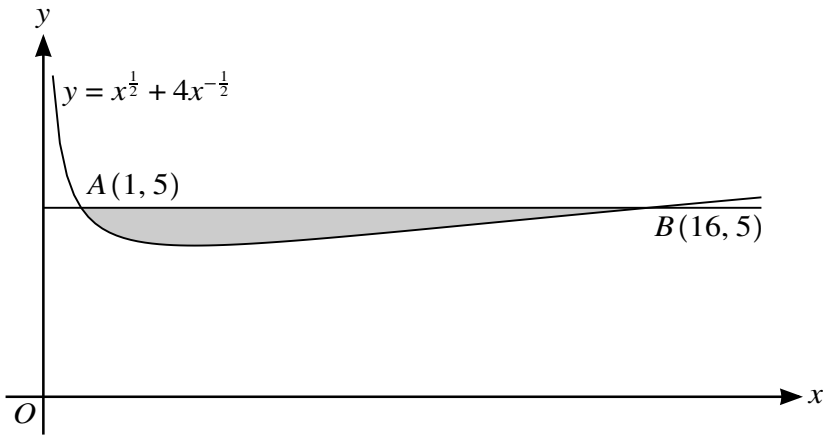



The diagram shows the circle with equation $(x - 2)^2 + (y + 4)^2 = 20$ and with centre $C$. The point $B$ has coordinates (0, 2) and the line segment $BC$ intersects the circle at $P$.

**(a)** Find the equation of $BC$. [2]

**(b)** Hence find the coordinates of $P$, giving your answer in exact form. [5]

**8**

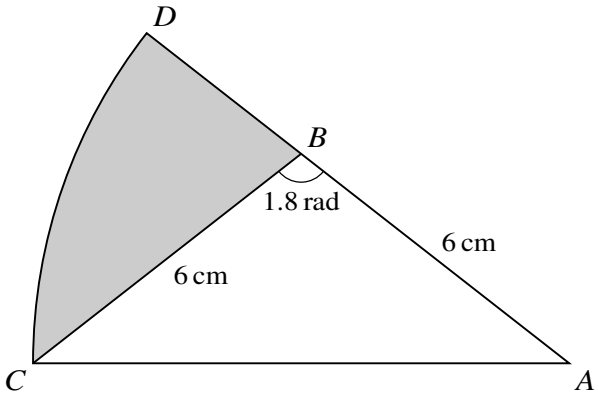



The diagram shows the curve with equation $y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$. The line $y = 5$ intersects the curve at the points $A\,(1, 5)$ and $B\,(16, 5)$.

**(a)** Find the equation of the tangent to the curve at the point $A$. [4]

**(b)** Calculate the area of the shaded region. [4]

**9**




The diagram shows triangle $ABC$ with $AB = BC = 6$ cm and angle $ABC = 1.8$ radians. The arc $CD$ is part of a circle with centre $A$ and $ABD$ is a straight line.

**(a)** Find the perimeter of the shaded region. [5]

**(b)** Find the area of the shaded region. [3]

**10** The function f is defined by $\mathrm{f}(x) = (4x + 2)^{-2}$ for $x > -\frac{1}{2}$.

**(a)** Find $\int_1^\infty \mathrm{f}(x)\,\mathrm{d}x$. [4]

A point is moving along the curve $y = f(x)$ in such a way that, as it passes through the point $A$, its $y$-coordinate is **decreasing** at the rate of $k$ units per second and its $x$-coordinate is **increasing** at the rate of $k$ units per second.

**(b)** Find the coordinates of $A$. [6]

**11** The point $P$ lies on the line with equation $y = mx + c$, where $m$ and $c$ are positive constants. A curve has equation $y = -\frac{m}{x}$. There is a single point $P$ on the curve such that the straight line is a tangent to the curve at $P$.

**(a)** Find the coordinates of $P$, giving the $y$-coordinate in terms of $m$. [6]

The normal to the curve at $P$ intersects the curve again at the point $Q$.

**(b)** Find the coordinates of $Q$ in terms of $m$. [4]

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